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ECE, EE, EEE, IN ENGINEERING

Theory
&
Objective

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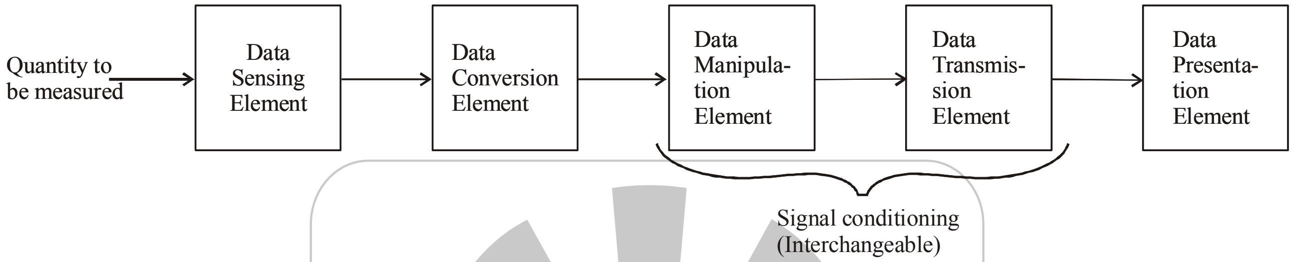
CONTENTS

1

GENERALIZED MEASURING SYSTEMS

THEORY

1.1 Block Diagram



1.1.1 Data Sensing Element

This stage is in direct contact with quantity under measurement. This element senses the quantity under measurement and produces some output suitable for the next stage. Output of sensing element or may not be an electrical signal.

Example : Transducer and sensors.

1.1.2 Data Conversion Element

This element converts one form of the signal to another form. The nature of the signal gets changed at output of this element.

Examples : A/D converter, D/A converter, V to I and I to V converter, V to f converter.

1.1.3 Data Manipulation Element

This element modifies or changes the level of the input signal without changing the nature of the signal.

Examples : Amplifiers, Modulators and Attenuators etc.

1.1.4 Data Transmission Element

This element provides the medium for transmission of data from the process to control panel.

Examples : All transmission channels like optical fiber, two wire line Co-axial cable.

1.1.5 Data Presentation Element

This element is used to display or store the data for future analysis.

Examples : Display devices, like-CRO, Recorders, Plotters ect.

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PRACTICE SHEET

OBJECTIVE QUESTIONS

1. Some of the functional building blocks of a measurement system are:

Primary sensing element (PSE)

Variable conversion element (VCE), or transducer

Data transmission element (DTE)

Variable Manipulation element (VME)

Data presentation element (DPE)

The correct sequential connection of the functional building blocks for an electronic pressure gauge will be

(a) PSE, VME, VCE, DPE, DTE

(b) PSE, VCE, VME, DTE, DPE

(c) DTE, DPE, VCE, PSE, VME

(d) PSE, VED, DTE, DPE, VME

2. Which of the following are data representation elements in a generalized measurement system?

1. Analog indicator
2. Amplifier
3. A/D converter
4. Digital display

Select the correct answer using the codes given below:

(a) 1 and 2 (b) 1 and 4

(c) 2 and 4 (d) 3 and 4

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ANSWERS AND EXPLANATIONS

1. ***Ans.(b)***

The correct sequential connection of the functional building blocks for an electronic pressure gauge will be:

PSE, VCE, VME, DTE, DPE

2. ***Ans.(b)***

Data representation elements in a generalized measurement system are

1. Analog indicators
2. Digital displays
3. Magnetic tapes
4. High speed camera

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2

STATIC & DYNAMIC CHARACTERISTICS OF MEASURING INSTRUEMENTS

THEORY

2.1 STATIC CHARACTERISTICS

2.1.1 Accuracy

It is the degree of closeness with which measured value approaches true value of quantity under measurement. Accuracy is a conformity to truth.

2.1.2 Precision

It is the reproducibility of the same result again and again. It is the measure of consistency in the results. Precision is measured with an index called precision index (h). The precision index describes the spread or dispersion of repeated result about a central value.

Note : Displays with higher significant figure have higher precision.

Example

(a) 105

(b) 105.0

(c) 0.00105×10^5

(a) and (c) - have three significant figure.

(b) - has four significant figure

So (b) has higher precision.

Note

(a) Accuracy of an instrument can be improved by re-calibrating the instrument but precision can not be improved because it is design time characteristic of the instrument.

(b) Precision is prerequisite of accuracy, A highly precised instrument may not be accurate but for high accuracy instrument must be precised.

2.1.3 Sensitivity

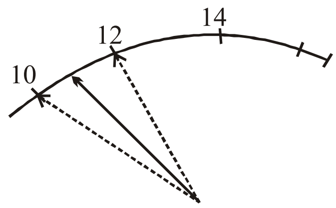
It is defined as the change in output of the instrument per unit change in input.

$$S = \frac{\Delta q_o}{\Delta q_i}$$

where, q_0 is output of instrument and q_i is input quantity under measurement.

2.1.4 Resolution

It is the smallest change in input quantity which can be measured with an instrument. Resolution is also known as discrimination.



Note **:** Resolution of an instrument can be improved by re-calibrating the scale but sensitivity cannot be improved because it is design-time characteristics.

2.1.5 Dead Zone

It is the range of input of an instrument for which output is zero.

Threshold : The last critical value of input within the dead zone for which output becomes non-zero is called threshold. It is the minimum input required to be applied to get the measurable output.

Difference between Resolution and Threshold : Resolution is the minimum change in input which can be measured where as threshold is the minimum input required to get measurable output.

2.1.6 Reproducibility

It is the degree of closeness with which a given value is repeatedly measured over a specified period of time. The perfect reproducibility means that the instruments has zero drift from the true value.

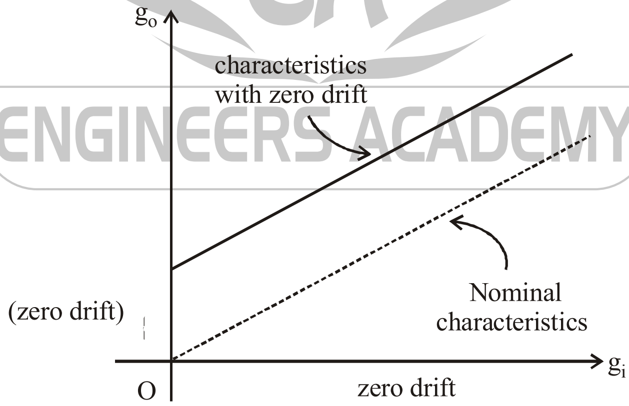
Repeatability : It is the degree of closeness with which a reading is repeated again and again in the given set of reading. Repeatability is the variation in scale reading and is random in nature.

2.1.7 Drift

Drift means variation in output of an instrument from the desired value for a given input.

2.1.7.1 Types of Drift

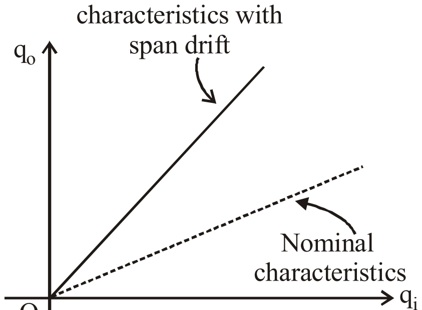
(a) Zero drift



Desired characteristics → ----------

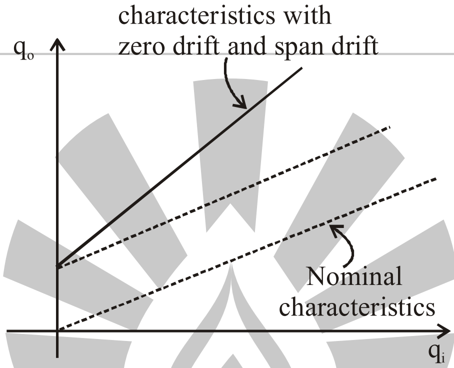
Actual characteristics → _______

(b) Sensitivity Drift



span or sensitivity drift

(c) Zonal Drift : If instrument exhibits both zero and sensitivity drift then deviation from desired characteristics of instrument is called zonal drift.



Zero drift and Span drift

***Note* :** Zero drift can be removed by re-calibrating the instrument but sensitivity and zonal drifts can't be removed because these are the design time characteristics.

2.2 DYNAMIC CHARACTERISTICS

It gives the behavior of the instrument with respect to time. The transient and steady state responses of the instrument depend on the dynamic characteristic.

Transfer function of 1st order and IInd order instruments.

Case - I

$$\frac{q_o}{q_i} = \frac{k}{s\tau + 1}$$

where, k is sensitivity constant and τ is time constant.

Case - II

$$\frac{q_o}{q_i} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

where, ξ is damping ratio and ω_n is natural undamped frequency.

Time constant of underdamped response of instrument,

Note **:** The dynamic response of indicating type instruments (e.g. Galvanometer) is always under damped with damping ratio from 0.7 to 0.8.

Time delay or time lag in case-I order instrument is given by :

$$T_d = \frac{\tan^{-1} \omega\tau}{\omega}$$

where, τ is time constant and ω is frequency.

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PRACTICE SHEET

OBJECTIVE QUESTIONS

1. Which one of the following defines resolution?

(a) The least interval between two adjacent discrete details, which can be distinguished from one another

(b) The largest interval between highest and lowest measured values, that the device can be adjusted to measure

(c) The least interval between highest and lowest measured values, that the device can be adjusted to measure

(d) The ratio of a change in output magnitude to the change in the input which causes it, after the steady state has been reached

2. Which one of the following is the best definition of accuracy?

(a) It is the measure of consistency or reproducibility of measurements

(b) It is the ratio of the change in output signal to the change in input signal

(c) It is the smallest change in measurable input

(d) It is the closeness with which an instrument reading approaches the true value of the quantity being measured

3. The precision of an instrument indicates its ability to reproduce a certain reading with a given

(a) drift (b) resolution

(c) shift (d) consistency

4. A first order instrument is characterized by

(a) Time constant only

(b) Static sensitivity and time constant

(c) Static sensitivity and damping coefficient

(d) Static sensitivity, damping coefficient and natural frequency of oscillations

5. Match List-I (Terms) with List-II (Its meaning) and select the correct answer using the codes given below the lists:

List-I

A. Precision

B. Accuracy

C. Resolution

D. Static sensitivity

List-II

1. The smallest change in the input quantity which can be tested with its certainty.
2. Closeness of the reading with its true value
3. Measure of reproducibility of the measurements.
4. Ratio of infinitesimal change in output to infinitesimal change input.

Codes:	**A**	**B**	**C**	**D**
(a)	2	3	1	4
(b)	3	2	4	1
(c)	3	2	1	4
(d)	2	3	4	1

6. To achieve the optimum transient response, the indicating instruments are so designed as to

(a) be critically damped

(b) be undamped

(c) provide damping which is slightly more than the critical value

(d) provide damping which is slightly less than the critical value

7. Consider the following statements:

In a measuring instruments,

1. Linearity is more important than sensitivity.
2. High precision indicates high accuracy.
3. Accuracy cannot be better than resolution.

Of these statements

(a) 1, 2 and 3 are correct

(b) 1 and 2 are correct

(c) 2 and 3 are correct

(d) 1 and 3 are correct

8. A resistance of 105 ohms is specified using significant figures as indicated below:

1. 105 ohms
2. 105.0 ohms
3. 0.000105 Megaohms.

Among these,

(a) 1 represents greater precision than 2 and 3

(b) 2 represents greater precision but 1 and 3 represent same precision

(c) 2 and 3 represent same precision

(d) None of these

9. A voltmeter has a uniform scale with 100 divisions. The full-scale reading is 10 V and could be read upto 1/10 of a scale division with some degree of certainty. It's resolution is

(a) 0.1 V (b) 0.02 V

(c) 0.01 V (d) 0.001 V

10. When a first order instrument with a time constant of 10 m sec is used to measure an input x = 10 sin 100 t. the time lag will be

(a) $\frac{\tan^{-1}1}{100}$sec (b) 100 $\tan^{-1}1$ sec

(c) $\frac{100}{\tan^{-1}1}$sec (d) $\tan^{-1}1$ sec

11. The following terms used in the context of an instrument are numbered as shown:

(1) accuracy, (2) sensitivity,

(3) precision and (4) resolution

Match these with their possible definitions listed below

P. Repeatability of readings on successive observations

Q. Smallest perceptible change in the output

R. Deviation of the output from the true value

S. Minimum value of the input from the true value

T. Ratio of the change in the instrument reading to the change in the measured variable.

(a) 1-P, 2-Q, 3-R, 4-S

(b) 1-S, 2-Q, 3-P, 4-T

(c) 1-R, 2-T, 3-P, 4-Q

(d) 1-T, 2-Q, 3-P, 4-R

12. The threshold of an instrument is normally defined

(a) as the smallest measurable input change (non-zero value) which can be detected

(b) as the smallest measurable input which can be detected

(c) in terms of linearity of scale

(d) as a function of drift

13. The desirable static characteristics of measuring systems are the following:

(a) Accuracy only

(b) Sensitivity only

(c) Reproducibility only

(d) All above

14. Absolute instruments are those:

(a) Which give the magnitude of the quantity under measurement in terms of physical constants of the instrument.

(b) Which give the magnitude of the quantity being measured by observing the output indicated by the instrument.

(c) Which are calibrated by comparison with a secondary instrument.

(d) Which are very commonly used.

15. Dead zone of an instrument is the:

(a) Difference between output and input

(b) Smallest input change that can be detected

(c) largest input change to which the output is zero

(d) value of output for zero input

16. ***Assertion (A):*** A precision instrument is always accurate.

Reason (R): A precision instrument is one where the degree of reproducibility of the measurements is very good

(a) Both A and R are individually true and R is the correct explanation of A

(b) Both A and R are individually true but R is not the correct explanation of A

(c) A is true but R is false

(d) A is false but R is true

17. Match List-I (Term) with List-II (Statement) and select the correct answer using the code given below the lists:

List-I

A. Relative error

B. Precision

C. Calibration

D. Resolution

List-II

1. The ability of the device to give identical output when repeated measurements are made with the same input signal
2. The ratio difference between measured value and the true value to the true value of the measureand
3. The smallest increment in measurand that can be detected with certainty by the instrument
4. The process of making adjustments on the scale so that the instrument readings conform to an accepted standard

Codes	**A**	**B**	**C**	**D**
(a)	2	3	4	1
(b)	4	1	2	3
(c)	4	3	2	1
(d)	2	1	4	3

18. For a first order instrument a 5% settling time is equal to

(a) three times the time constant

(b) two times the time constant

(c) the time constant

(d) time requried for the output signal to reaches 5% of the final value

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ANSWERS AND EXPLANATIONS

1. ***Ans.(a)***

Resolution is the least interval between two adjacent discrete details, which can be distinguished from one another.

2. ***Ans.(d)***

Accuracy is the closeness with which an instrument reading approaches the true value of the quantity being measured.

3. ***Ans.(d)***

The precision of an instrument indicates its ability to reproduce a certain reading with a given consistency.

4. ***Ans.(b)***

The general transfer function of a first order instrument is given by,

$$G(s) = \frac{S}{1+\tau s}$$

where, S = static sensitivity

τ = time constant

5. ***Ans.(c)***

A. Precision is measure of reproducibility of the measurements.

B. Accuracy is closeness of the reading with its true value.

C. Resolution is the smallest change in the input quantity which can be de tested with its certainty.

D. Static sensitivity is the ratio of infinitesimal change in output to infinitesimal change input.

6. ***Ans.(d)***

To achieve the optimum transient response, the indicating instruments are so designed as to provide damping which is slightly less than the critical value.

7. ***Ans.(d)***

High precision does not guarantee high accuracy.

Therefore, statement '2' is incorrect.

8. ***Ans.(b)***

A resistance of 105 ohms is specified using significant figures as indicated below:

1. 105 ohms
2. 105.0 ohms
3. 0.000105 megohms.

Among these 105.0 ohms has 4 significant figures and 105 ohms and 0.000105 megaohms has 3 significant figures each. Therefore statement 2 represents greater precision but statements 1 and 3 represent same significant figures.

9. ***Ans.(c)***

$$\text{One scale division} = \frac{\text{full scale reading}}{\text{number of division}}$$

$$\Rightarrow \text{One scale division} = \frac{10}{100} = 0.1$$

$$\text{Resolution} = \frac{1}{10} \times \text{ scale division}$$

$$\Rightarrow \text{Resolution} = \frac{1}{10} \times 0.1\text{V} = 0.01\text{V}$$

10. ***Ans.(a)***

For 1st order systems, time lag

$$= \frac{1}{\omega}\tan^{-1}\omega\tau$$

$$\Rightarrow \text{Time lag} = \frac{1}{100}\times\tan^{-1}(100 \times 10 \times 10^{-3})\text{sec}$$

$$\Rightarrow \text{Time lag} = \frac{\tan^{-1}}{100}\text{sec}$$

11. ***Ans.(c)***

(1) accuracy is deviation of the output from the true value.

(2) sensitivity is ratio of the change in the instrument reading to the change in the measured variable.

(3) precision is repeatability of readings on successive observations

(4) resolution is smallest perceptible change in the output.

12. ***Ans.(b)***

The threshold of an instrument is normally defined as the smallest measurable input which can be detected.

13. ***Ans.(d)***

The desired characteristics of a measuring system are accuracy, sensitivity and reproducibility. The undesired characteristics are drift, static errors and dead zone.

14. ***Ans.(a)***

Absolute instruments are those which give the magnitude of the quantity under measurement in terms of physical constants of the instrument.

15. ***Ans.(c)***

Dead zone of an instrument is the largest input change to which the output is zero.

16. ***Ans.(d)***

I. The high precision instruments are not necessarily accurate. Therefore, the assertion is false.

II. A precision instrument is one where the degree of reproducibility of the measurements is very good The reason is true.

17. ***Ans.(d)***

A. Relative error is defined as the ratio difference between measured value and the true value to the true value of the measureand.

B. Precision is defined as the ability of the device to give identical output when repeated measurements are made with the same input signal.

C. Calibration is defined as the process of making adjustments on the scale so that the instrument readings conform to an accepted standard.

D. Resolution is defined as the smallest increment in measurand that can be detected with certainty by the instrument.

18. ***Ans.(a)***

For a first order instrument 5% settling time is equal to three times the time constant and 2% settling time is equal to four times the time constant

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3

ERROR ANALYSIS

THEORY

3.1 Classification of Errors

3.1.1 Absolute Error

The deviation between measured value and true value of the quantity under measurement is called absolute error. The absolute error is also known as Limiting or Guarantee Error. Mathematically absolute error is given by,

$$\delta A = A_m - A_T$$

Where A_m is measured value or actual value

and A_T is true value or nominal value of the quantity.

3.1.2 Relative Error

The absolute error as fraction of true quantity is called relative error.

$$e_r = \frac{\delta A}{A_T} = \frac{A_m - A_T}{A_T}$$

Percentage Error/Percent Relative Error

$$\%\text{ Relative error} = \%\varepsilon_r = \frac{\delta A}{A_T} \times 100$$

$$= \%\varepsilon_r = \frac{A_m - A_T}{A_T} \times 100$$

Note

(i) The percentage limiting error (accuracy) of measuring instrument is always given in terms of full scale reading.

(ii) Limiting error of a measuring instrument remains fixed irrespective of reading of the instrument, e.g. A voltmeter of 0 - 150 V with 2% accuracy \will have an absolute error of 3 volt (= 0.02 × 150 V) irrespective of its reading. In other words it will give an error of 3 volt for any reading say 75V, 50V, 25V etc.

Example : A voltmeter of range 0 - 150V has an accuracy of 2%. Find the % error when the instrument reads :

(i) 100 volt (ii) 75 volt (iii) 50 volt.

Solution : Absolute error $\delta A = 0.02 \times 150$

$\delta A = 3$ volt

(i) Error at 100 volt, $\% \text{ error} = \frac{3}{100} \times 100 = 3\%$

(ii) Error at 75 volt, $\% \text{ error} = \frac{3}{75} \times 100 = 4\%$

(iii) Error at 50 volt, $\% \text{ error} = \frac{3}{50} \times 100 = 6\%$

Settling Time(T_S) : It is the time required to reach the output with in 2 % to 5 % tolerance band.

$$T_s = 4t; \text{ for } 2\% \text{ band}$$
$$= 3\tau; \text{ for } 5\% \text{ band}$$

Note **:** Percentage error at any reading 'X' of an instrument can be given by,

$$\% \text{ error at any reading} = \left(\frac{\text{Full scale reading}}{X}\right) \times (\% \varepsilon_r \text{ at full scale})$$

Errors in reading of voltmeters of above example can be given by,

(i) % error at 100 volt, $\% \ \varepsilon_r = \frac{150}{100} \times 2 = 3\%$

(ii) % error at 75 volt, $\% \ \varepsilon_r = \frac{150}{75} \times 2 = 4\%$

(iii) % error at 50 volt, $\% \ \varepsilon_r = \frac{150}{50} \times 2 = 6\%$

3.1.3 Known and Unknown Errors

Known Errors : These error can be determined and **true value** of measured quantity can be determined by adding the error to the measured value. Known error is always either positive or negative.

Unknown Errors : These errors may have positive or negative sign. Therefore, it can not be determined exactly but these error are generally given in terms of maximum deviation from true value. For example: A resistance of $100\Omega \pm 2\Omega$ has a maximum deviation of $\pm 2\Omega$ and the resistance may lie any where in between 98Ω to 102Ω.

3.2 Combination of Quantities with Limiting Errors

3.2.1 Sum of Quantities

Let $X = X_1 + X_2 + \ldots\ldots\ldots\ldots + X_n$

$$\Rightarrow \quad \pm\partial X = \pm(\partial X_1 + \partial X_2 + \ldots\ldots + \partial X_n)$$

$$\Rightarrow \quad \pm\frac{\partial X}{X} = \pm\left(\frac{\partial X_1}{X} + \frac{\partial X_2}{X} + \ldots\ldots + \frac{\partial X_n}{X}\right)$$

$$\Rightarrow \quad \pm\frac{\partial X}{X} = \left(\frac{\partial X_1}{X_1} \cdot \frac{X_1}{X} + \frac{\partial X_2}{X_2} \cdot \frac{X_2}{X} \ldots + \frac{\partial X_n}{X_n} \cdot \frac{X_n}{X}\right)$$

Example : Two resistances of 100Ω ± 1% and 200Ω ± 3 % are connected in series. Find the % error and rage of equivalent resistance of the combination.

Solution

$$R = R_1 + R_2$$

$$R = 100 + 200 = 300\Omega$$

$$\frac{\partial R}{R} = \pm\left(\frac{\partial R_1}{R_1}.\frac{R_1}{R} + \frac{\partial R_2}{R_2}.\frac{R_2}{R}\right)$$

$$\frac{\partial R}{R}\times 100\% = \left(\frac{\partial R_1}{R_1}\times 100\%\right)\frac{R_1}{R} + \left(\frac{\partial R_2}{R_2}\times 100\%\right)\frac{R_2}{R}$$

$$= \left[1\times\frac{100}{300} + 3\times\frac{200}{300}\right]$$

$$\frac{\partial R}{R}\times 100\% = \frac{700}{300}\% = \pm 2.33\%$$

$$\partial R = \pm\left(\frac{700}{300}\times\frac{1}{100}\right)\times 300 = \pm 7\Omega$$

Range of R = R ± ∂R = 300 ± 7Ω

= 293 Ω to 307 Ω

3.2.2 Difference of Quantities

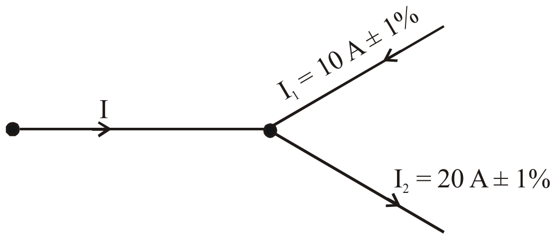
Let $X = X_1 - X_2$

For unknown error $\partial X = \pm(\partial X_1 + \partial X_2)$

$$\Rightarrow \quad \frac{\partial X}{X} = \pm\left(\frac{\partial X_1}{X} + \frac{\partial X_2}{X}\right)$$

$$\Rightarrow \quad \frac{\partial X}{X} = \pm\left(\frac{\partial X_1}{X_1}.\frac{X_1}{X} + \frac{\partial X_2}{X_2}.\frac{X_2}{X}\right)$$

Example : Find using error analysis ?



Solution : Applying KCL,

$$I_1 + I = I_2$$

$\therefore$ $I = I_2 - I_1 = 20 - 10 = 10A$

$$\Rightarrow \quad \frac{\partial I}{I} = \pm\left(\frac{\partial I_2}{I_2}.\frac{I_2}{I} + \frac{\partial I_1}{I_1}.\frac{I_1}{I}\right)$$

$$\Rightarrow \quad \frac{\partial I}{I} = \pm\left(1\times\frac{20}{10} + 1\times\frac{10}{10}\right)$$

$$\Rightarrow \quad \frac{\partial I}{I} = \pm 3\%$$

$\therefore$ $I = 10A \pm 3\%$

3.2.3 Multiplication of Quantities

Let $X = X_1 \ X_2 \ .... \ X_n$

Taking log on both sides, we get,

$$\log X = \log X_1 + \log X_2 + ... + \log X_n$$

Differentiating w.r.t. X,

$$\therefore \quad \frac{1}{X} = \frac{1}{X_1}\frac{\partial X_1}{\partial X} + \frac{1}{X_2}\frac{\partial X_2}{\partial X} + ... + \frac{1}{X_n}\frac{\partial X_n}{\partial X}$$

$$\Rightarrow \quad \frac{\partial X}{X} = \frac{\partial X_1}{X_1} + \frac{\partial X_2}{X_2} + ... + \frac{\partial X_n}{X_n}$$

For unknown errors
$$\frac{\partial X}{X} = \pm\left(\frac{\partial X_1}{X_1} + \frac{\partial X_2}{X_2} + ... + \frac{\partial X_n}{X_n}\right)$$

Example : A current of 10A ± 2% is passed through a resistance of 100Ω ± 1Ω. Then find the error and range of voltage measured across the resistor.

Solution

$$V = IR = 10 \times 100 = 1000 \text{ volt}$$

$$\therefore \quad \frac{\partial V}{V} = \pm\left(\frac{\partial R}{R} + \frac{\partial I}{I}\right)$$

$$\Rightarrow \quad = \pm\left(\frac{1}{100}\times 100 + 2\right)\%$$

$$\Rightarrow \quad \frac{\partial V}{V} = \pm 3\%$$

$$\Rightarrow \quad \partial V = \pm\frac{3}{100}\times 1000$$

$$\therefore \quad \partial V = \pm 30 \text{ volt}$$

Range of V = (1000 – 30) to (1000 + 30)

$\Rightarrow$ Range of V = 970 to 1030 volt.